

# Departure Time Choices in Traffic Congestion

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# Introducing myself

- Takamasa Iryo, Dr. Eng.  
Assistant Professor, Kobe University,  
working with Prof. Asakura.
- Studied in the University of Tokyo till 2002  
and got a degree under the supervision of  
Prof. Kuwahara.
- Main topic: Transport Network Analysis,  
esp., DTA / Pedestrians

# Kitamura sensei in my study

- Kobe is near to Kyoto.
- Easy to visit Kyoto.
- I had many chances to have discussions with “Sensei” and his staffs and students.



# Today's talk

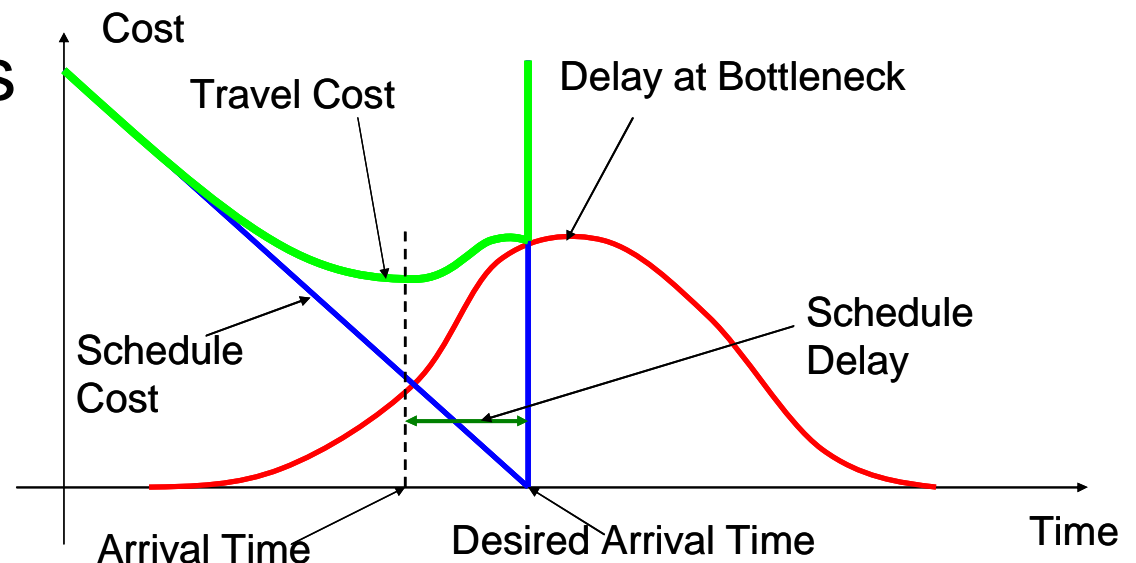
- Departure Time Choices:  
in the context of the dynamic traffic assignment
- Equilibrium Approach:  
A joint work with Prof. Yoshii at Kyoto University.
- Non-equilibrium Approach

# Behaviour vs. Network

- Modelling traveller's **behaviour** of departure time choices: Behavioural Study (e.g. Small(1982))
- Modelling traveller's **interactions** through departure time choices under congestion. Network Assignment Study (e.g. Vickrey(1969))

# Time choice under congestion

- Define a schedule cost function.  
(bi-linear or non-linear)
- Define travel cost function, which is the sum of schedule cost and delay cost at a bottleneck.
- Assume travellers choose the departure time with the lowest cost in a day.



# Equilibrium or Non-equilibrium?

- [Equilibrium]  
Find traveller's departure time choice pattern satisfying **Wardrop's first principle** within a day:  
Looking for a temporal equilibrium point
- [Non-equilibrium]  
Construct a day-to-day model and see change of traveller's behaviour over days:  
Simulating a **day-to-day dynamics** of travellers

# Equilibrium

- Many existing studies: e.g. Smith(1984), Daganzo(1985), Newell(1985), Arnott et al. (1993 and others), Lindsey(2004).
- My contribution with Prof. Yoshii
  - There is an optimization problem that is equivalent to the equilibrium condition.
  - Total schedule cost of travellers is minimized in equilibrium.
  - The model is applicable to the more general situation; “single-bottleneck-per-route”.

Iryo, T. and Yoshii, T., Equivalent Optimization Problem for Finding Equilibrium in the Bottleneck Model with Time Choices, in *Mathematics in Transport*, B.G. Heydecker, ed., Elsevier: Oxford, p. 231-244, 2007.



Traveller's choice  
(route and arrival time)

Schedule cost  
(incl. detour cost)

Minimize

$$\sum_{\text{all } i, j} X_{(i,j)} T_{(i,j)}$$

Subject to:

$$\sum_{\text{all } j} X_{(i,j)} \leq A(i) \quad \text{Capacity constraint}$$

$$\sum_{\text{all } i} X_{(i,j)} = D(j) \quad \text{Demand constraint}$$

$$X_{(i,j)} \geq 0 \quad \text{Non-Negative Constraint}$$

The KKT condition of the optimization problem is equivalent to the Wardrop's first principle.

Maximize  $\sum_{i=1}^n D(i) \theta_i - \sum_{i=1}^n w(i) \left( \frac{T(i)}{\Phi} - T_0 \right)$

subject to  $w(i) \geq 0$

$w(i) + \frac{T(i)}{\Phi} - T_0 \leq 0$  for all  $i$

Delay at the bottleneck

Travel cost

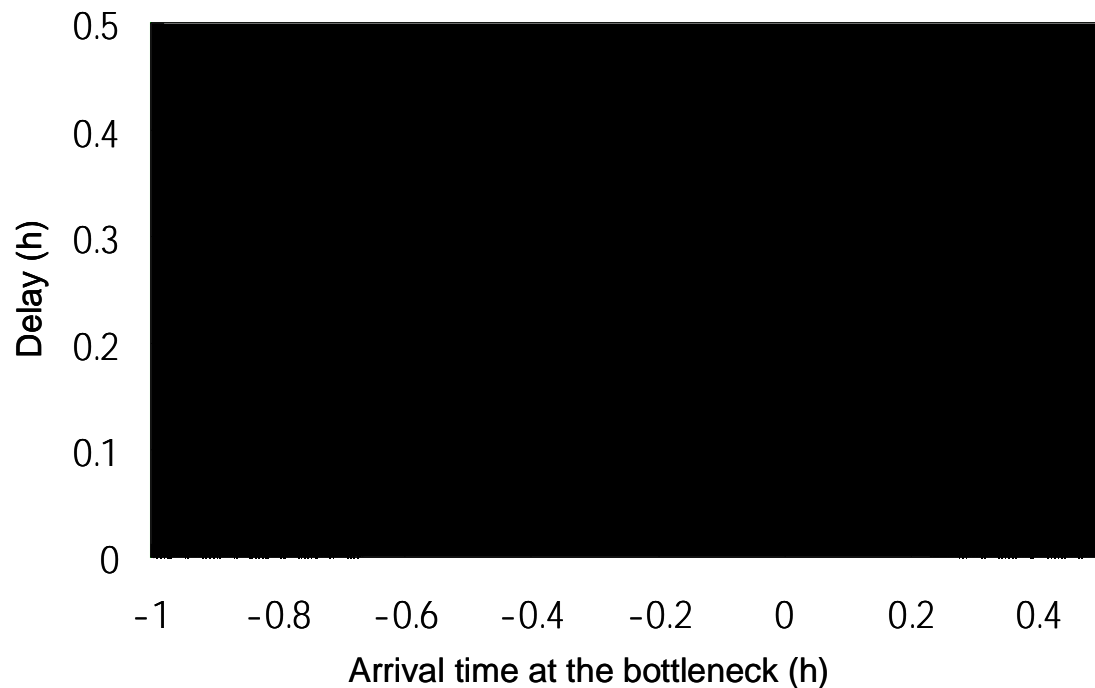
The dual problem can solve delays at bottlenecks; delays are the dual variable of the primal problem.

# Day-to-day dynamics

- A simple day-to-day dynamics is constructed after Smith(1984) and Mounce(2006).
- Travellers move towards a choice with smaller cost gradually.
- A system converges to equilibrium if travel cost function is monotone increase against number of travellers; it is normal in traffic assignments which is not dynamic.

# Day-to-day dynamics

- Monotonicity of travel cost is not guaranteed where travellers are faced to a schedule constraint, especially when they try to arrive on time.
- Behaviour of travellers do not converge to equilibrium in some cases.



Iryo, T., An Analysis of Instability in a Departure Time Choice Problem, *Journal of Advanced Transportation*, 42(3), p. 333-356, 2008.

# From system to behaviour

- Equilibrium: Simple.  
Good characteristic (equivalent to optimization)
- Equilibrium is not stable, may not be achieved??
- What is the meaning of equilibrium?  
How travellers behave under non-equilibrium situation?
- Behavioural viewpoint might be demanded to think of the meaning of equilibrium.